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THEORY OF THE MAGNETIC BOUNDARY LAYER*

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In this work, examples are cited which illustrate the phenomenon that a moving plasma is shielded from an external magnetic field and from the electric currents that flow in it; the thickness of the shielding layer, called a magnetic boundary layer, has the order $1/\sqrt{Re_m}$ for motions at large magnetic Reynolds numbers.

1) If we introduce the vector potential of the magnetic field $\vec{W}$ $(\vec{H} = \text{curl}\, \vec{W})$ then the equations of magneto-hydrodynamics can be put in the form

$$\frac{\partial \vec{W}}{\partial t} - \vec{V} \times \text{curl}\, \vec{W} = -\text{grad}\, q + \nu_m \nabla^2 \vec{W} \;;\quad \text{div}\, \vec{W} = 0$$

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{V}) = 0 \;;\quad \rho \frac{dV_\alpha}{dt} = -\frac{\partial p_m}{\partial x_\alpha} + \frac{1}{4\pi} H_\beta \frac{\partial H_\alpha}{\partial x_\beta} + \frac{\partial \tau_{\beta\alpha}}{\partial x_\beta}$$

$$\rho T \frac{ds}{dt} = \tau_{\alpha\beta} \frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial}{\partial x_\alpha}\left(k \frac{\partial T}{\partial x_\alpha}\right) + \frac{\nu_m}{4\pi} j^2 \;;\quad p = f(\rho; s)$$

where $\vec{V}$ is the velocity vector with components V_α $(\alpha = 1, 2, 3)$; $-\text{grad}\, q = \vec{E} + \frac{1}{c}\frac{\partial \vec{W}}{\partial t}$; $\vec{E}$ is the electric field strength; ν_m is the magnetic viscosity, which in the general case is a function of the temperature T and the pressure p ; ρ is the density of the medium; $p_m = p + \frac{H^2}{8\pi}$; $\vec{H}$ is the magnetic field strength; $\tau_{\alpha\beta}$ is the viscous stress tensor; s is the entropy of unit mass; k is the coefficient of thermal conductivity; $\vec{j}$ is the vector current density. The first equation of the system as written resembles the equation of hydrodynamics in Lamb's form. The electrodynamical part of the equations of magneto-hydrodynamics in their customary form

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can be obtained from the first equation of the system if one applies to it the curl operation and sets $\nu_m = const$ in the flow.

2) Let there exist a semi-infinite plate (coinciding with the half-plane xz, $x > 0$) along which there flows an electric current in the direction Oz, and let the plate be immersed in a conducting fluid at rest, the current in the plate being insulated from the fluid.

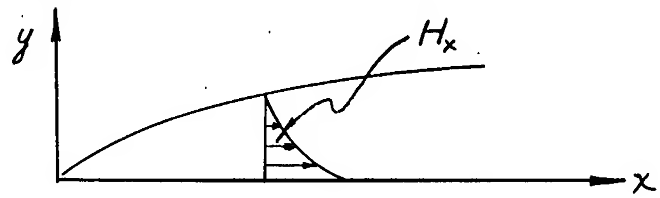

Fig. 1

It is easy to see that there is introduced in the fluid a magnetic field denoted by the vector $\vec{H}$, parallel to the plane xy. We now bring the fluid into motion at velocity $\vec{U}$ in the direction Ox, and we consider regions of the flow, sufficiently removed from the edge of the plate in the direction of the x-axis, so that the corresponding magnetic Reynolds number $Re_m = UL/\nu_m$ will be a large quantity; then, on the basis of the preservation of vector lines in a medium of infinite conductivity, and on the strength of the requirement $Re_m \gg 1$, the magnetic field vanishes in the main flow and persists only in the layer adjacent to the surface of the plate and having a thickness of order $L/\sqrt{Re_m}$. We call this layer the magnetic boundary layer of the first kind.

If we consider again the problem of a semi-infinite plate in a stream of conducting fluid for the case $Re_m \gg 1$, where this time a current flows on the plate in the direction of the x-axis with a constant linear (along Oz) intensity (the plate is insulated from the outer flow with the exception of the leading edge, and regions removed from the beginning of the plate), then, on the basis of the properties of the preservation of vector lines $\vec{j}$ (2), the electric current in the fluid is localized in a layer adjacent to the plate and having a thickness of order $L/\sqrt{Re_m}$. We call this layer a magnetic boundary layer of the second kind.

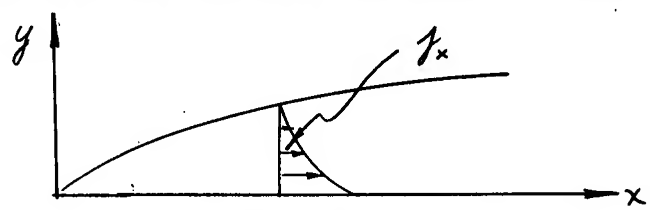

Fig. 2.

3) Carrying out, in the equations of paragraph 1), estimates similar to those made concerning the ordinary boundary layer (see, for example, (1)) for the magnetic boundary layer of the first kind, we obtain the following equations:

$$\frac{\partial W}{\partial t} + u\frac{\partial W}{\partial x} + v\frac{\partial W}{\partial y} = \nu_m \frac{\partial^2 W}{\partial y^2}$$

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p_m}{\partial x} + \frac{1}{4\pi}\left(\frac{\partial W}{\partial y}\frac{\partial^2 W}{\partial x\,\partial y} - \frac{\partial W}{\partial x}\frac{\partial^2 W}{\partial y^2}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial p_m}{\partial y} = 0\ ;\ \rho T\left(\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y}\right) = \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\nu_m}{4\pi}\left(\frac{\partial^2 W}{\partial y^2}\right)^2$$

where W is the z - component of the vector $\vec{W}$; μ is the coefficient of the ordinary viscosity.

Analogously, the equations of the magnetic boundary layer of the second kind take the form:

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \nu_m \frac{\partial^2 H}{\partial y^2}$$

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p_m}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\ ;\quad \frac{\partial p_m}{\partial y} = 0$$

$$\rho T\left(\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y}\right) = \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\nu_m}{4\pi}\left(\frac{\partial H}{\partial y}\right)^2$$

In arriving at the equations for magnetic boundary layers of the first and second kinds, terms have been retained, in the equations of paragraph 1) whose order relative to those omitted is $1/Re_m$.

As is clear from the equations of magnetic boundary layers, the pressure across them can change by many times (since the quantity $p + H^2/8\pi = const$ across the layer), something which markedly distinguishes the boundary layers under consideration from the ordinary ones. This circumstance, existing as a consequence of the electro-magnetic field forces, can be applied in questions of thermally insulated bodies: it is sufficient to make $H^2/8\pi\big|_{y=0} > p_\infty$ (see, for example, the two articles (2)), since near the plate there appears a zone of cavitation, i.e., a zone, where $p \approx 0$.

In the case of an incompressible fluid, the equations of the steady magnetic boundary layer of the first kind take the form:

$$\frac{D(W;\psi)}{D(x;y)} = \nu_m\frac{\partial^2 W}{\partial y^2}\ ;\ \frac{D\left(\frac{\partial\psi}{\partial y};\psi\right)}{D(x;y)} - \frac{1}{4\pi\rho}\frac{D\left(\frac{\partial W}{\partial y};W\right)}{D(x;y)} = -\frac{1}{\rho}\frac{\partial p_m}{\partial x} + \nu\frac{\partial^3\psi}{\partial y^3}$$

while the equations for the steady boundary layer of the second kind are:

$$\frac{D(H;\psi)}{D(x;y)} = \nu_m \frac{\partial^2 H}{\partial y^2} \quad ; \quad \frac{D\left(\frac{\partial\psi}{\partial y};\psi\right)}{D(x;y)} = -\frac{1}{\rho}\frac{\partial p_m}{\partial x} + \nu \frac{\partial^3\psi}{\partial y^3}$$

where ψ is the stream function, ν the kinematic viscosity coefficient.

4) For the equations of the magnetic boundary layer of the first kind for an incompressible fluid, there exists the following class of similar solutions:

$$\psi = x^\beta \nu_m^{1/2} U^{1/2} f(\zeta) \quad ; \quad W = \sqrt{4\pi\rho\,\nu_m U}\; x^\gamma g(\zeta)$$

$$\zeta = \nu_m^{-1/2} U^{1/2} x^\alpha y \quad ; \quad p_m = \delta q x^\delta + p_{mo}$$

where $\alpha, \beta, \gamma, q, \delta, p_{mo}$ are certain constants.

The functions f and g satisfy the system of ordinary differential equations:

$$\gamma g f' - \beta g' f = g'' ; \ (\alpha+\beta) f'^2 - \beta f f'' - (\alpha+\gamma) g'^2 + \gamma g g'' = \delta^2 q + \omega f'''$$

Here $\omega = \nu/\nu_m$, $\alpha = (\delta-2)/4$, $\beta = \gamma = (\delta+2)/4$. In the case where $p_m = const$, $\gamma = \beta = \alpha + 1$.

For the equations of the magnetic boundary layer of the second kind for an incompressible fluid, the analogous class of similar solutions takes the form:

$$\psi = \nu_m^{1/2} U^{1/2} x^\beta f(\zeta) \quad ; \quad H = \nu_m^{1/2} x^\gamma h(\zeta)$$

$$\zeta = U^{1/2} \nu_m^{-1/2} x^\alpha y \quad ; \quad p_m = \delta q x^\delta + p_{mo}$$

where the functions $f(\zeta)$ and $h(\zeta)$ satisfy the equations:

$$\gamma h f' - \beta h' f = h'' \quad ; \ (\alpha+\beta) f'^2 - \beta f f'' = \delta^2 q + \omega f'''$$

Here $\alpha = (\delta-2)/4$, $\beta = (\delta+2)/4$, γ is an arbitrary constant; if $p_m = const$, $\beta = \alpha + 1$.

5) In the classes of solutions mentioned in paragraph 4) solutions are to be found for problems of the flow over semi-infinite plates in the presence of a magnetic boundary layer of both the first and second kinds, if the x - component of the magnetic field $H_x = const = H_0$ along the plate.

In the case of the magnetic boundary layer of the first kind, the problem comes down to the solution of the system of equations: $(\alpha = -1/2 ; \beta = \gamma = 1/2 ; \delta = 0)$:

$$g f' - g' f = 2 g'' \ ; \ g g'' - f f'' = 2\omega f''' \qquad \left(\zeta = y \Big/ \sqrt{\frac{\nu_m x}{U}}\right)$$

with the following boundary conditions:

a) $\zeta = 0 : \ f = f' = 0 \ ; \ g' = \dfrac{H_0}{\sqrt{4\pi\rho}\, U}$

b) $\zeta \to \infty : f' \to 1, \ g' \to 0$

If $\omega \ll 1$, that is, if the processes associated with the influence of the ordinary viscosity are non-existent, then conditions a) are replaced by:

$$f = 0 \quad , \quad g' = \frac{H_0}{\sqrt{4\pi\rho}\,U} \quad \text{for } \zeta = 0$$

In the case of the magnetic boundary layer of the second kind, the problem comes down to the solution of the Blasius equation: $(\alpha = -\tfrac{1}{2}, \beta = \tfrac{1}{2}, \gamma = \delta = 0)$:

$$-\tfrac{1}{2} f f'' = \omega f'''$$

and the integral

$$h(\zeta) = C_1 + C_2 \exp \int_0^\zeta \exp\left(-\tfrac{1}{2}\int_0^h f(t)\,dt\right) d\zeta$$

The boundary conditions take the form:

a) $\zeta = 0$: $f = f' = 0$; $h = H_0 \nu_m^{-1/2}$

b) $\zeta \to \infty$: $f' \to 1$, $h \to 0$

If $\omega \ll 1$, then $f = \zeta$ and $h = H_0 \nu_m^{-1/2}\left[1 - erf(\zeta/2)\right]$, where $erf(x)$ is the error function ((4), p. 120).

In the class of solutions of paragraph 4) are also contained the solutions corresponding to the flow over bodies in the presence of a magnetic boundary layer, with the magnetic field given on its internal boundary and the velocity given on the external boundary, according to a power law.

6) In the case of the magnetic boundary layer of the first kind the electric field is equal to zero and the Joule heating appears at the expense of the energy of the main stream; in this case there appear additional resistive forces (besides the usual ones of viscous friction). In the case of the magnetic boundary layer of the second kind, the electric field, on the other hand, is the principal factor; Joule heating appears due to the work of the external e.m.f.s (a propos of this, see also (3)).

The examples considered show that a moving plasma tends to be shielded from an external magnetic field and from the electric currents that flow in it - this analogously to the known effect of shielding of a plasma at rest from an external electric field.

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